Aspects of nuclear electroweak processes in EFTs

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Contents

- Brief reviews on
  - Aspects on electro-weak currents of HBChPT
  - Challenges we have encountered
- Discussions

Meson-exchange currents :1970's

- $\nabla \cdot \vec{J} = -i\, [H, \rho(x)]$,
  H contains $V_\tau = V\, \tau_1 \cdot \tau_2$, $\rho(x)$ contains τ_{1z} & τ_{2z},
  and $[\tau_1 \cdot \tau_2,\ \tau_{1z}] \sim \tau_1 \times \tau_2$

=> there should be meson-exchange currents (MEC)

=> N, Δ, π, ρ, ω, σ, a_1, …

- Chemtob & Rho derived MEC using current algebra
- Riska & Brown considered the $\sigma(np \rightarrow d\,\gamma)$, trying to remove the gap between exp= 330 mb, 1B= 300 mb.

Heavy-Baryon chiral perturbation theory: 1990's

- With only N & π
  - Minimal set, respecting the "spontaneously broken chiral symmetry", the most important aspect of low-energy QCD
  - Expansion parameter = Q/Λ_χ,
    Q = typical momentum transfer and /or m_π,
    $\Lambda_\chi \sim m_N \sim 4\pi f_\pi \sim 1$ GeV .
  - Other dofs $(\rho,\ \omega,\ \Delta,\ \cdots)$: integrated out
  - Well-defined power-counting, systematic & consistent expansion => "EFT" became an everyday language

Power counting

- $\nu = 2 - \left(\frac{E_N}{2} + E_{ext}\right) + 2\,L + \sum_i \bar{\nu_i},\ \ \bar{\nu_i} = \left(d_i + \frac{n_i}{2} + e_i\ - 2\right)$

Contributions to the nuclear current at $\mathbf{q} = 0$

Park et. al. PRC 67, 055206 (2003)

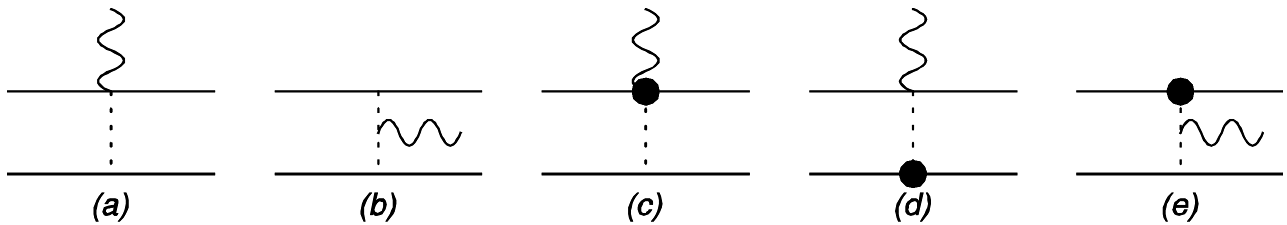
J_μ	LO	NLO	N^{2}LO	N^{3}LO	N^{4}LO
A	1B	-	1B-RC	2B	1B-RC, 2B-1L, and 3B
A_0	-	1B	2B	1B-RC	1B-RC, 2B-1L
V	-	1B	2B	1B-RC	1B-RC, 2B-1L
V_0	1B	-	-	2B	1B-RC, 2B-1L, and 3B

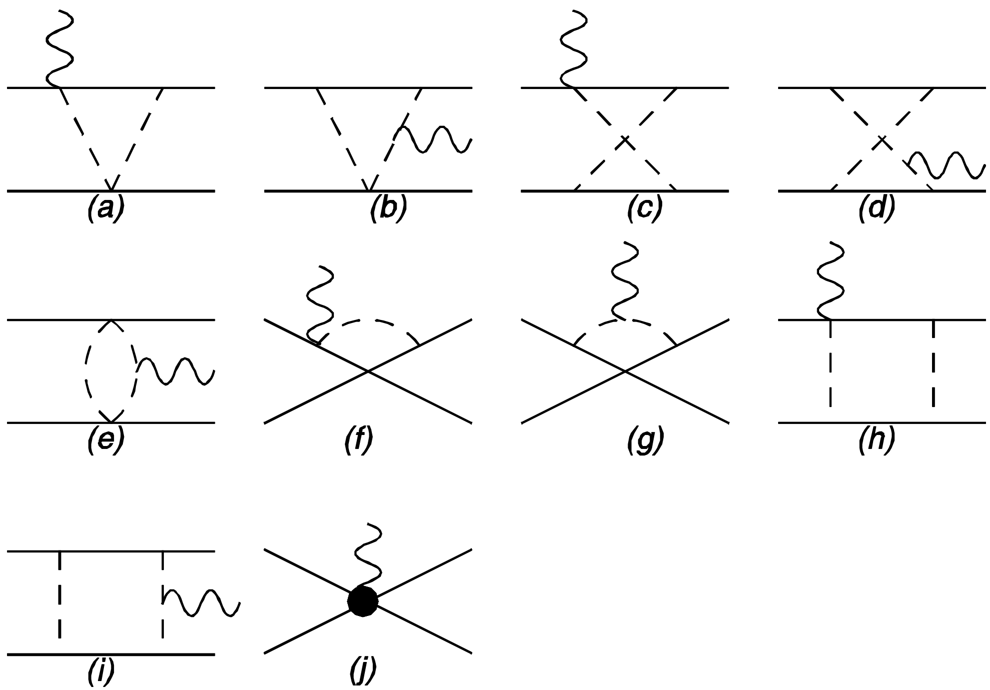
- With heavy mesons

$$\nu = 2\ - \left(\frac{E_N}{2} + E_H + E_{ext}\right) +\ 2\,L + \sum_i \left(d_i + \frac{n_i}{2} + h_i + e_i\ - 2\right)$$

2B vector currents up to N^3LO

$$J = J_{1B} + J_{1\pi} + (J_{1\pi C} + J_{2\pi} + J_{CT}) = LO + NLO + N^3LO$$





$$\boldsymbol{\mu}_{1\mathrm{B}}(q) = \sum_i \frac{1}{2m_p}\Big\{\hat{j}_0(qr_i)\left[\boldsymbol{\sigma}_i\left(\mu_i - Q_i\frac{\bar{\boldsymbol{p}}_i^2}{2m^2}\right) - \frac{\mu_i - Q_i}{2m^2}\bar{\boldsymbol{p}}_i\boldsymbol{\sigma}_i\cdot\bar{\boldsymbol{p}}_i\right]$$
$$+ \hat{j}_1(qr_i)\left[Q_i\boldsymbol{r}_i\times\bar{\boldsymbol{p}}_i\left(1-\frac{\bar{\boldsymbol{p}}_i^2}{2m^2}\right) - \frac{w(2\mu_i - Q_i)}{4m}i\boldsymbol{r}_i\times(\bar{\boldsymbol{p}}_i\times\boldsymbol{\sigma}_i)\right]$$
$$+ \frac{(qr_i)^2}{30}\hat{j}_2(qr_i)(3\hat{\boldsymbol{r}}_i\hat{\boldsymbol{r}}_i\cdot\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_i) + \cdots\Big\} \quad (2)$$

$$\boldsymbol{\mu}_{2\mathrm{B}}(q) = \sum_{i<j}\left[\boldsymbol{\mu}_{ij}^{1\pi} + \left(\boldsymbol{\mu}_{ij}^{1\pi C} + \boldsymbol{\mu}_{ij}^{1\pi:fixed} + \boldsymbol{\mu}_{ij}^{2\pi} + \boldsymbol{\mu}_{ij}^{CT}\right)\right] = \mathrm{NLO} + \mathrm{N^3LO}. \quad (3)$$

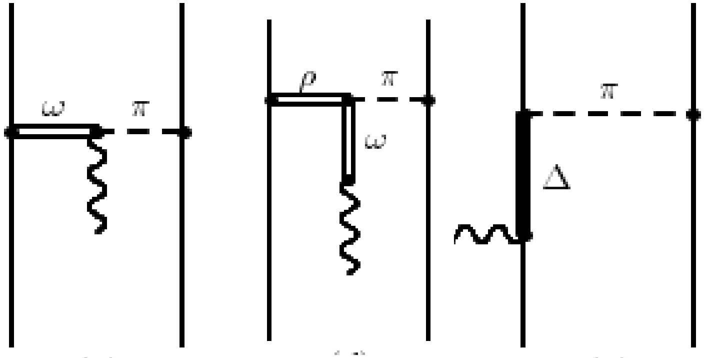
The leading MEC due to the *soft*-one-pion-exchange (1π) is NLO and given as

$$\boldsymbol{\mu}_{12}^{1\pi} = \frac{g_A^2}{8f_\pi^2}\left[\hat{T}_S^{(\times)}\left(\frac{2}{3}y_{1\Lambda}^{\pi}(r) - y_{0\Lambda}^{\pi}(r)\right) - \hat{T}_T^{(\times)}y_{1\Lambda}^{\pi}(r)\right]\hat{j}_0(qR)$$
$$- \frac{g_A^2 m_\pi^2}{24f_\pi^2}\tau_\times^z\boldsymbol{R}\times\boldsymbol{r}\left[\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\bar{y}_{0\Lambda}^{\pi}(r) + S_{12}y_{2\Lambda}^{\pi}(r)\right]\hat{j}_1(qR) + \cdots, \quad (4)$$

$$\boldsymbol{\mu}_{12}^{1\pi C} = -\frac{g_A^2}{8f_\pi^2}(\bar{c}_\omega + \bar{c}_\Delta)\left[(\hat{T}_S^{(+)} + \hat{T}_S^{(-)})\frac{\overline{y}_{0\Lambda}^{\pi}}{3} + (\hat{T}_T^{(+)} + \hat{T}_T^{(-)})\, y_{2\Lambda}^{\pi}\right]\hat{j}_1(qR)$$
$$+\frac{g_A^2}{8f_\pi^2}\bar{c}_\Delta\left[\frac{1}{3}\hat{T}_S^{(\times)}\overline{y}_{0\Lambda}^{\pi} - \frac{1}{2}\hat{T}_T^{(\times)}y_{2\Lambda}^{\pi}\right]\hat{j}_1(qR)$$
$$-\frac{1}{16f_\pi^2}\bar{N}_{WZ}\tau_1\cdot\tau_2\left[\boldsymbol{\sigma}_+\overline{y}_{0\Lambda}^{\pi} + (3\hat{\boldsymbol{r}}\hat{\boldsymbol{r}}\cdot\boldsymbol{\sigma}_+ - \boldsymbol{\sigma}_+)y_{2\Lambda}^{\pi}\right]\hat{j}_1(qR).$$

$$\boldsymbol{\mu}_{12}^{2\pi} = \frac{1}{128\pi^2 f_\pi^4}\left[\left(\hat{T}_S^{(+)} - \hat{T}_S^{(-)}\right)L_S + \left(\hat{T}_T^{(+)} - \hat{T}_T^{(-)}\right)L_T\right]\hat{j}_1(qR)$$
$$- \frac{1}{256\pi^2 f_\pi^4}(\tau_1\times\tau_2)^z\boldsymbol{R}\times\hat{\boldsymbol{r}}\frac{d}{dr}L_0\hat{j}_0(qR)$$

- The c_ω, c_Δ, N_{WZ} are LECs for πγNN vertex, where experimental data at low-E are poor.
- We used the resonance (ρ, ω, Δ) saturation assumptions to estimate their values



(a) (b)

$$\boldsymbol{\mu}_{12}^{CT} = \frac{1}{2m_p}[g_{4S}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) + g_{4V}T_S^{(\times)}]\delta_\Lambda^{(3)}(\boldsymbol{r})\hat{j}_0(qR)$$

$\sigma(np \rightarrow d\,\gamma)$

- 1% accuracy !
  - ChPT : 334(3) mb
  - Exp: 332.6(6) mb
- Why r_C –dependent ?
  - LEC (g_{4v}) for CT is set to 0, don't how to fix it.
- Can we do better ?
  - No, unless we can fix the LEC
  - Going higher order does not help
  - Including heavy dofs might be helpful

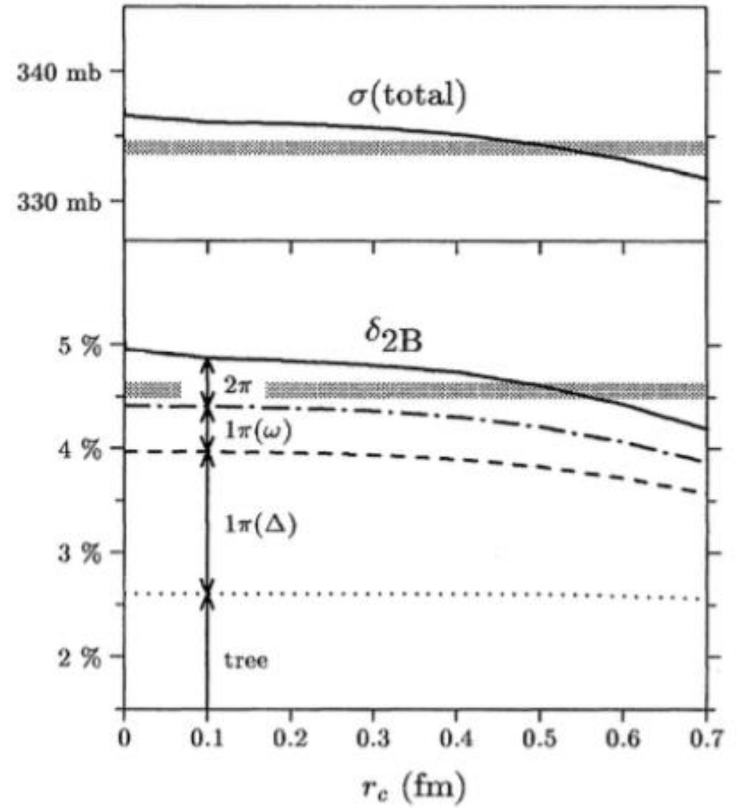


FIG. 2. Total capture cross section σ (top) and δ's (bottom) vs the cutoff r_c. The solid line represents the total contributions and the experimental values are given by the shaded band indicating the error bar. The dotted line gives δ_{tree}, the dashed line $\delta_{tree} + \delta_{1\pi}^{\Delta}$, the dot-dashed line $\delta_{tree} + \delta_{1\pi} = \delta_{tree} + \delta_{1\pi}^{\Delta} + \delta_{1\pi}^{\omega}$, and the solid line the total ratio δ_{2B}.

Renormalization &RG invariance

- Contact terms represent the contributions from the integrated-out dofs
- Total = (long + short) + CT
- RG invariance : short-ranged contributions are well represented by local operators

  (short-range) = $C_0\ \delta(r_{ij}) + C_2\ \partial^2\delta(r_{ij}) + \ldots$
  - Correspondence
    - (short-range) = model-dependent part
    - C_0 , C_2, ... = LECs
  - So, mistakes/mismatches in the short-range region means only different values of C_0 , C_2, ..., while the net results remain invariant
  - Can be numerically checked by looking at Λ-dependence.
- But RG invariance does not help for any long-ranged mismatches

$$\mu_{12}^{CT} = \frac{1}{2m_p}[g_{4S}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) + g_{4V}T_S^{(\times)}]\delta_\Lambda^{(3)}(r)\hat{j}_0(qR)$$

- g_{4S} and g_{4V} appear in
  - $\mu(^2H)$, $\mu(^3H)$, $\mu(^3He)$, …
  - Cross sections/spin observables of $np \to d\gamma$, $nd \to t\gamma$, …
- We can fix g_{4S} and g_{4V} by
  - A=2 sector: $\mu(^2H)$ and $\sigma(np \to d\gamma)$
  - A=3 sector: $\mu(^3H)$ and $\mu(^3He)$
  - …

Λ-depdendence (model: INOY)

Fixed g_{4s} & g_{4v} by using $\mu(^3H)$ and $\mu(^3He)$

Λ [MeV]	μ(d)	σ(np) [mb]	σ (nd) [mb]
500	0.8584	330.9	0.501
600	0.8584	330.7	0.497
700	0.8585	330.5	0.495
800	0.8583	330.5	0.495
900	0.8583	330.4	0.496
Exp.	0.8574	332.6(6)	0.508(15)

but …

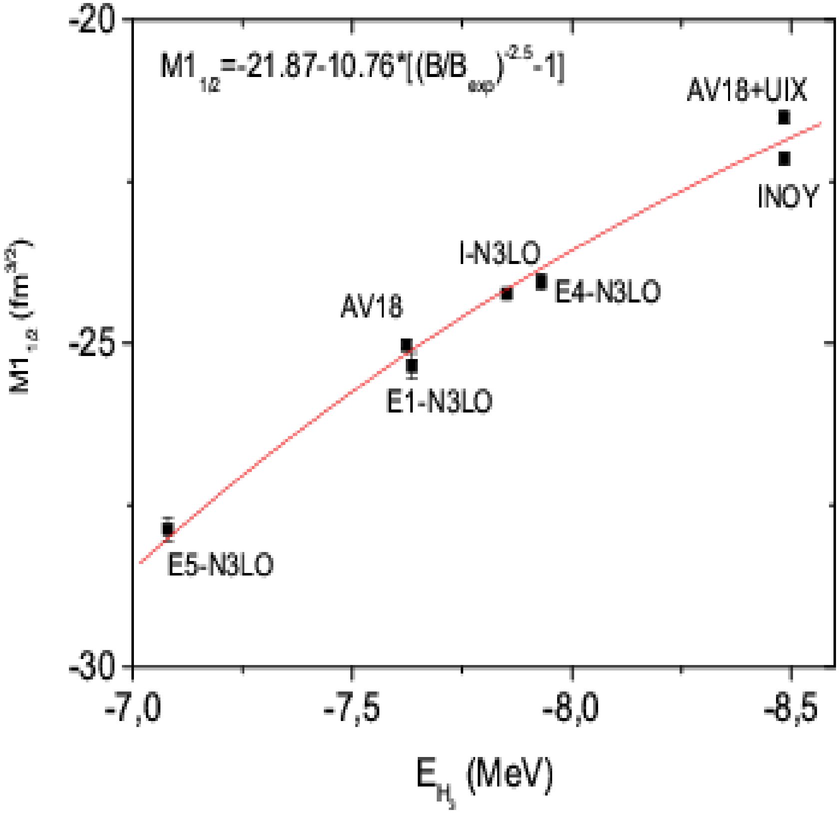
Model-dependence (naïve version)

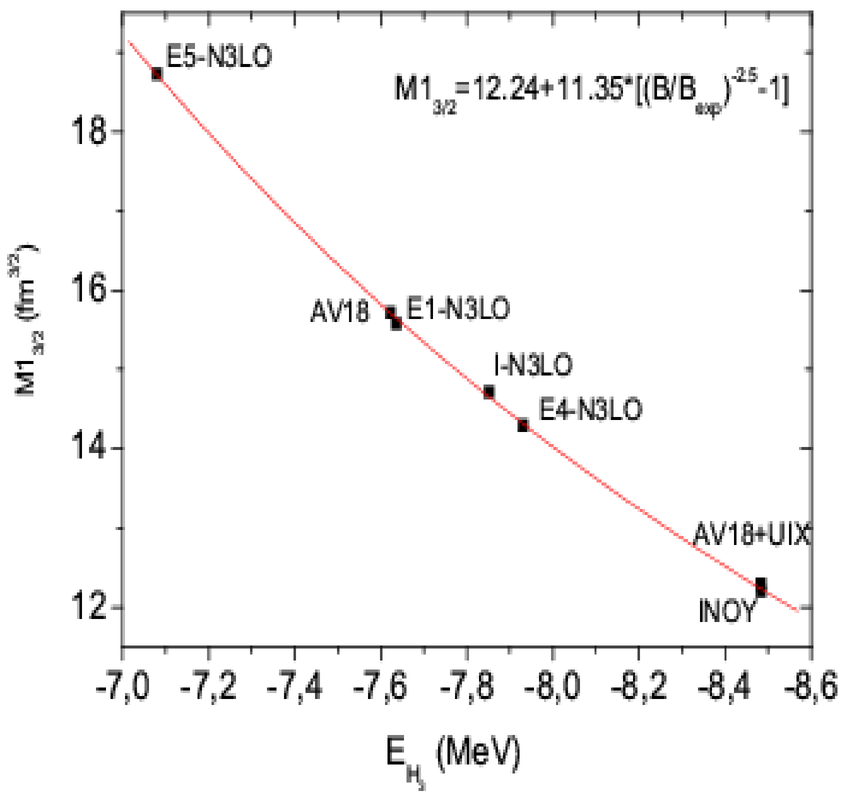
Model	**μ(d)**	**σ(np) [mb]**	**σ (nd) [mb]**	**BE(H3) [MeV]**	**BE(He3) [MeV]**
Av18	0.858	331.9	**0.68**	7.623	6.925
Av18+U9	0.860	330.6	**0.48**	8.483	7.753
INOY	0.859	330.6	**0.50**	8.483	7.720
I-N3LO	0.857	330.4	**0.63**	7.852	7.159
E1-N3LO	0.858	328.7	**0.69**	7.636	6.904
E4-N3LO	0.859	331.0	**0.61**	7.930	7.210
E5-N3LO	0.855	330.9	**0.88**	7.079	6.403
Exp.	0.8574	332.6(7)	**0.58(1)**	8.482	7.718

Issue : What if potential fails to reproduce ERPs accurately

- Mismatches in the long-range region
- RG invariance says nothing here, and we are in trouble !
- Possible solutions :
  - 1. Use only accurate potentials
  - 2. Adjust the potential to reproduce those ERPs correctly
  - 3. Find out the correlation between matrix elements and ERPs

M1(J=1/2) ≈ $\phi_2(B_3)$, M1(J=3/2) ≈ $\phi_4(B_3)$





$$\phi_2(B_3) = -21.87 - 10.76\left[(B_3/B_3^{\exp})^{-2.5} - 1\right],$$
$$\phi_4(B_3) = 12.24 + 11.35\left[(B_3/B_3^{\exp})^{-2.5} - 1\right].$$

Model-dependence (improved)

Model	μ_d	σ_np [mb]	σ_nd [mb]	BE(H3) [MeV]	BE(He3) [MeV]
Av18+U9	0.860	330.6	**0.478(3)**	8.483	7.753
INOY	0.859	330.6	**0.498(3)**	8.483	7.720
I-N3LO+U9*	**0.859(1)**	**330.2(4)**	**0.479(4)**	**8.482**	**7.737**
Exp.	0.8574	332.6(7)	**0.508(15)**	8.482	7.718

- Model-dependence is tricky
  - Naively large model-dependence
  - But strongly correlated to the triton binding energy
  - Model-independent results could be obtained
    - either by making use of the correlation curve
    - or by adjusting the parameters of 3N potential to have correct ERPs

The *hen* process ($^3He + n \rightarrow {}^4He + \gamma$)

$\sigma(\text{exp}) = (55 \pm 3)\ \mu b,\ (54 \pm 6)\ \mu b$

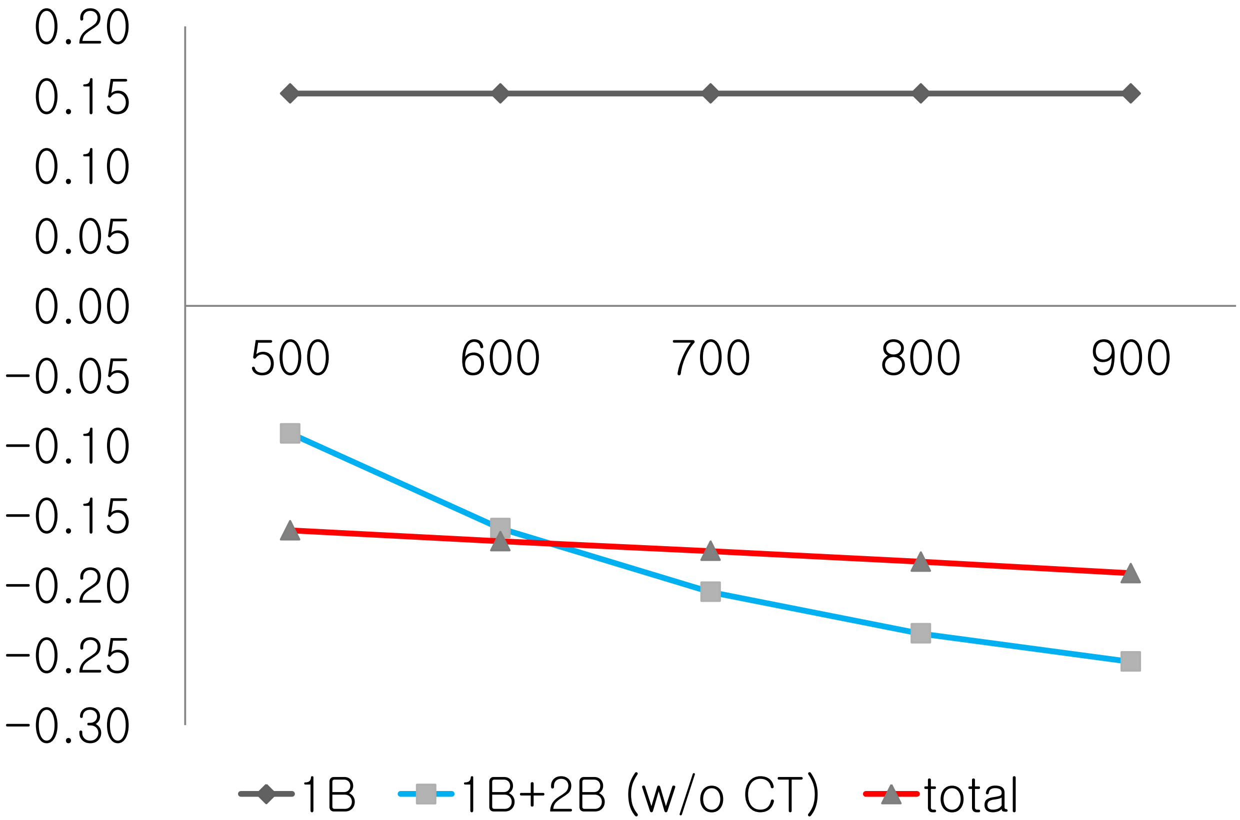
2-14	μb :	(1981) Towner & Kanna
50	μb :	(1991) Wervelman
(112, 140)	μb :	(1990) Carlson et al
(86, 112)	μb :	(1992) Schiavilla et al

Our calculation

- Difficulties
  - $< 1B\ (LO) > \rightarrow N^3LO$ due to pseudo-orthogonality between wfs
  - Coincidental cancellation between 1B and MEC occurs.
  - 4-body wave functions

- Wave functions: Faddeev-Yakubovski equations
- M1 currents up to N^3LO
- g_{4s} and g_{4v} : fixed by (^3H) and μ(^3He)

- *Cutoff-dependence*: (Re M_{hen}) vs Λ[MeV]

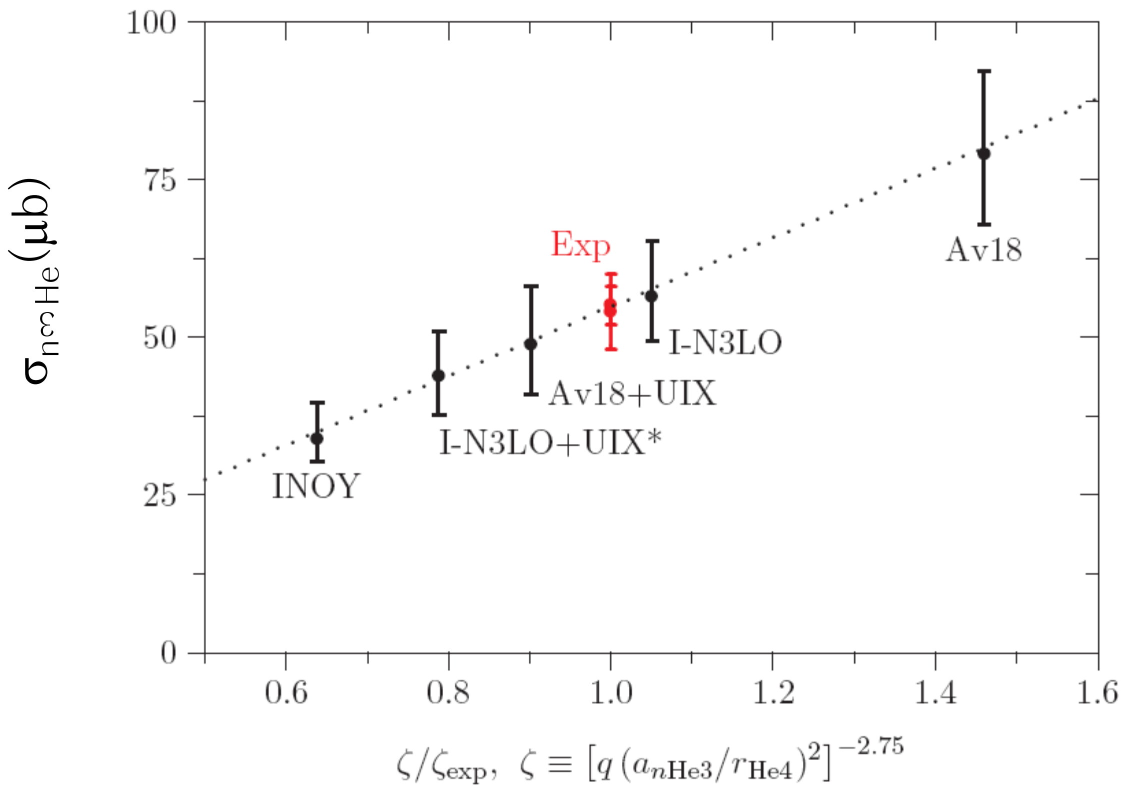


- *hen cross section*: model-dependent (naïve version)

Model	**σ_{n3He} [μb]**	**B (^{4}He) [MeV]**	**B (^{3}H) [MeV]**	**B (^{3}He) [MeV]**	**$^3a_{n3He}$ [fm]**	**r_{He4} [fm]**	**P_D(^{4}He) [%]**
Av18	80(12.2)	24.23	7.623	6.925	3.43-0.0082i	1.516	13.8
I-N3LO	57.3(7.9)	25.36	7.852	7.159	3.56-0.007i	1.520	9.30
I-N3LO+UIX*	**44.4(6.7)**	**28.12**	**8.482**	**7.737**	**3.44-0.0055i**	**1.475**	**10.9**
Av18+UIX	**49.4(8.5)**	**28.47**	**8.483**	**7.753**	**3.23-0.0054i**	**1.431**	**16.0**
INOY	34.4(4.5)	29.08	8.482	7.720	3.26-0.0058i	1.377	5.95
Exp.	**55(3); 54(6)**	28.3	8.482	7.718	3.28(5)-0.001(2)i	1.475(6)	

- *How hen cross section is correlated to ERPs ?*

$\sigma_{n3He} \propto \zeta \equiv [q\,(a_{nHe3}/r_{He4})^2]^{-2.75}$ or $\sigma_{n3He} \propto q^{-5} P_D^{2/3}$



Model	σ_{n3He} [μb] $\times\, \zeta_{exp}/\zeta$
Av18	**54.8(8.4)**
I-N3LO	**54.5(7.5)**
I-N3LO+UIX*	**56.4(8.5)**
Av18+UIX	**54.8(9.4)**
INOY	**53.9(7.5)**
Exp.	**55(3); 54(6)**

Convergence ?

TABLE V: Matrix elements for the Av18+UIX wave function with $\Lambda = 700$ MeV; the LEC values corresponding to this case are: $(g_{4s},\, g_{4v}) = (0.581,\, -0.4615)\,[\mathrm{fm}^3]$.

	$\mu(^3\mathrm{H})$	$\mu(^3\mathrm{He})$	$\Re\mathcal{M}$	$\Im\mathcal{M}$
1B: LO	2.5727	−1.7632	0.0964	−0.0136
1B: RC	−0.0171	0.0037	0.0554	−0.0075
1B-total	2.5556	−1.7595	0.1518	−0.0211
2B: 1π (NLO)	0.2292	−0.2258	−0.1657	0.0195
2B: $1\pi C$ (N^3LO)	0.1578	−0.1289	−0.1465	0.0172
2B: 2π (N^3LO)	0.0419	−0.0408	−0.0445	0.0052
finite (total w/o CT)	2.9845	−2.1550	−0.2049	0.0208
	$0.0193\, g_{4s}$	$0.0190\, g_{4s}$	$0.0205\, g_{4s}$	$-0.0014\, g_{4s}$
	$+0.0363\, g_{4v}$	$-0.0354\, g_{4v}$	$-0.0377\, g_{4v}$	$+0.0044\, g_{4v}$
2B: CT (N^3LO)	$= -0.0055$	$= 0.0274$	$= 0.0293$	$= -0.0029$
Total	2.9790	−2.1276	−0.1756	0.0179

“***Things Are Not What They Appear***”, Disney Pocahontas 2

- Observation: If we omit something (say, $<J_{2\pi}>$), values of g_{4s} and g_{4v} should also be changed to have the correct $\mu(^3H)$ and $\mu(^3He)$ without $<J_{2\pi}>$.

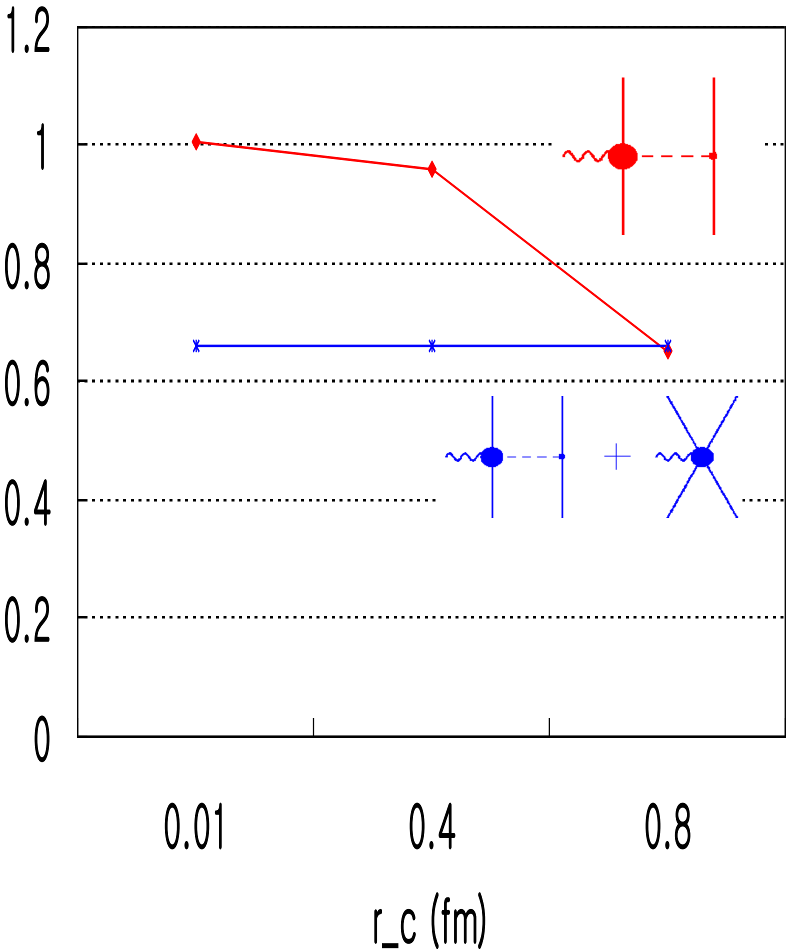
- “effective $<J_{2\pi}>$” ≡ changes in the net ME if we omit $<J_{2\pi}>$

	naïve <J>	**effective <J>**
2B: 1π (NLO)	-0.1657	0.0749
2B: 1πC(N^3LO)	-0.1465	-0.0093
2B: 2π (N^3LO)	-0.0445	-0.0010

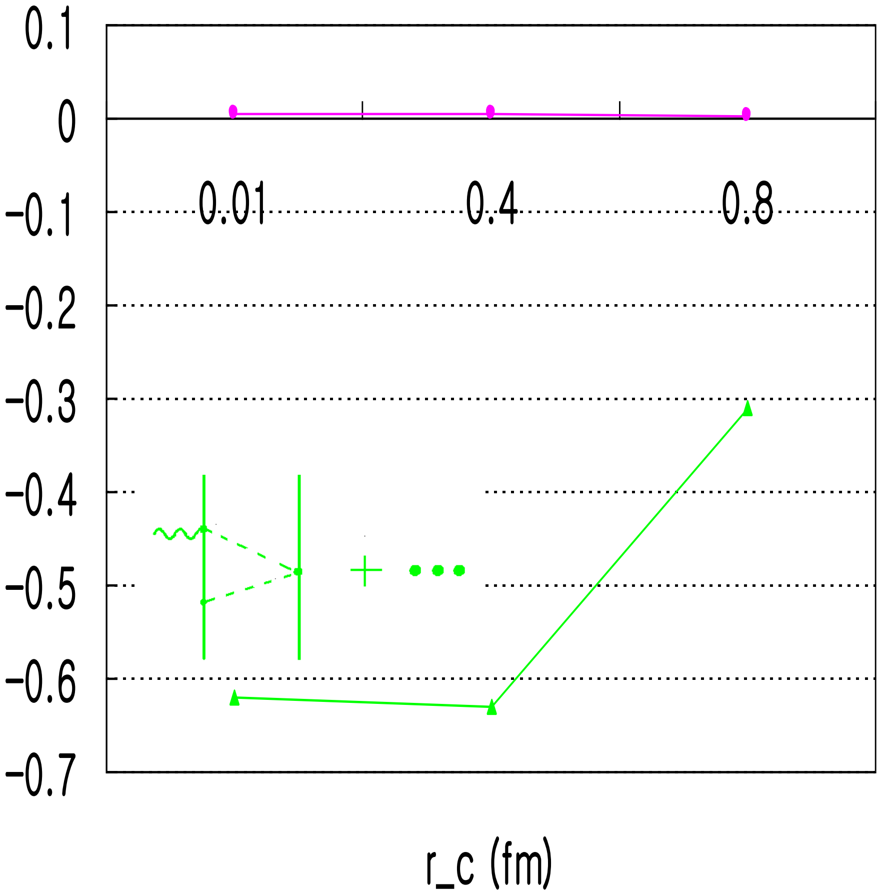
- If we omit 2π (N^3LO), our results will change only about 0.6 %, instead of 25% !

Results(M_{2B}/M_{1B}) of M1S ($n+p \rightarrow d+\gamma$) :

N^3LO



N^4LO

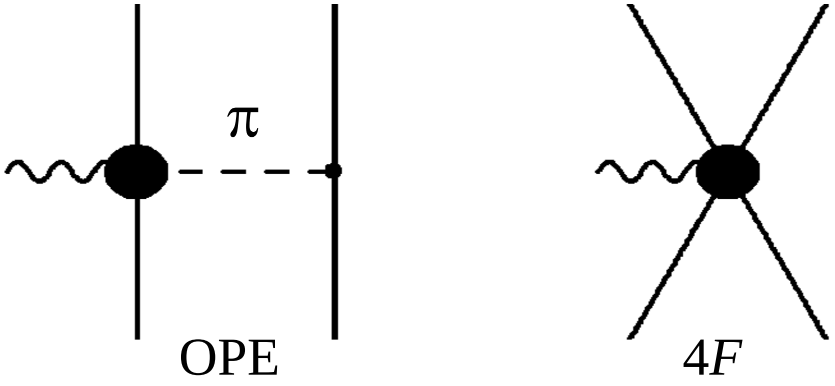


TSP, K. Kubodera, D.-P. Min & M. Rho, _PLB472('00)232

Gamow-Teller channel (*pp* and *hep*)

$$\vec{A}_{1B} = g_A \sum_i \tau_i \left[\vec{\sigma}_i + \frac{\overrightarrow{p_i \sigma_i \cdot p_i} - \vec{\sigma}_i {p_i}^2}{2m_N^2} \right] = \mathrm{LO} + \mathrm{N}^2\mathrm{LO}$$

$$\vec{A}_{2B} = \sum_{i<j} \left[\vec{A}_{ij}^{\mathrm{OPE}} + \vec{A}_{ij}^{4F} \right] = \mathrm{N}^3\mathrm{LO}$$



There is no soft-OPE (which is NLO) contributions

$$\vec{A}_{ij}^{4F} = -\frac{g_A}{m_N f_\pi^2}\left[2\hat{d}_1(\tau_i\vec{\sigma}_i + \tau_j\vec{\sigma}_j) + \hat{d}_2(\tau_i \times \tau_j)(\vec{\sigma}_i \times \vec{\sigma}_j)\right]$$

Thanks to Pauli principle and the fact that the contact terms are effective only for L=0 states, only one combination is relevant:

$$\hat{d}^R \equiv \hat{d}_1 + 2\hat{d}_2 + \frac{1}{3}\hat{c}_3 + \frac{2}{3}\hat{c}_4 + \frac{1}{6}$$

The same combination enters into

pp, *hep*, tritium-β decay (TBD),

μ-d capture, *ν–d* scattering, … .

We use the experimental value of TBD to fix $\hat{d}^R$,
then all the others can be predicted !

Results($p\,p \to d e^+ \nu_e$)

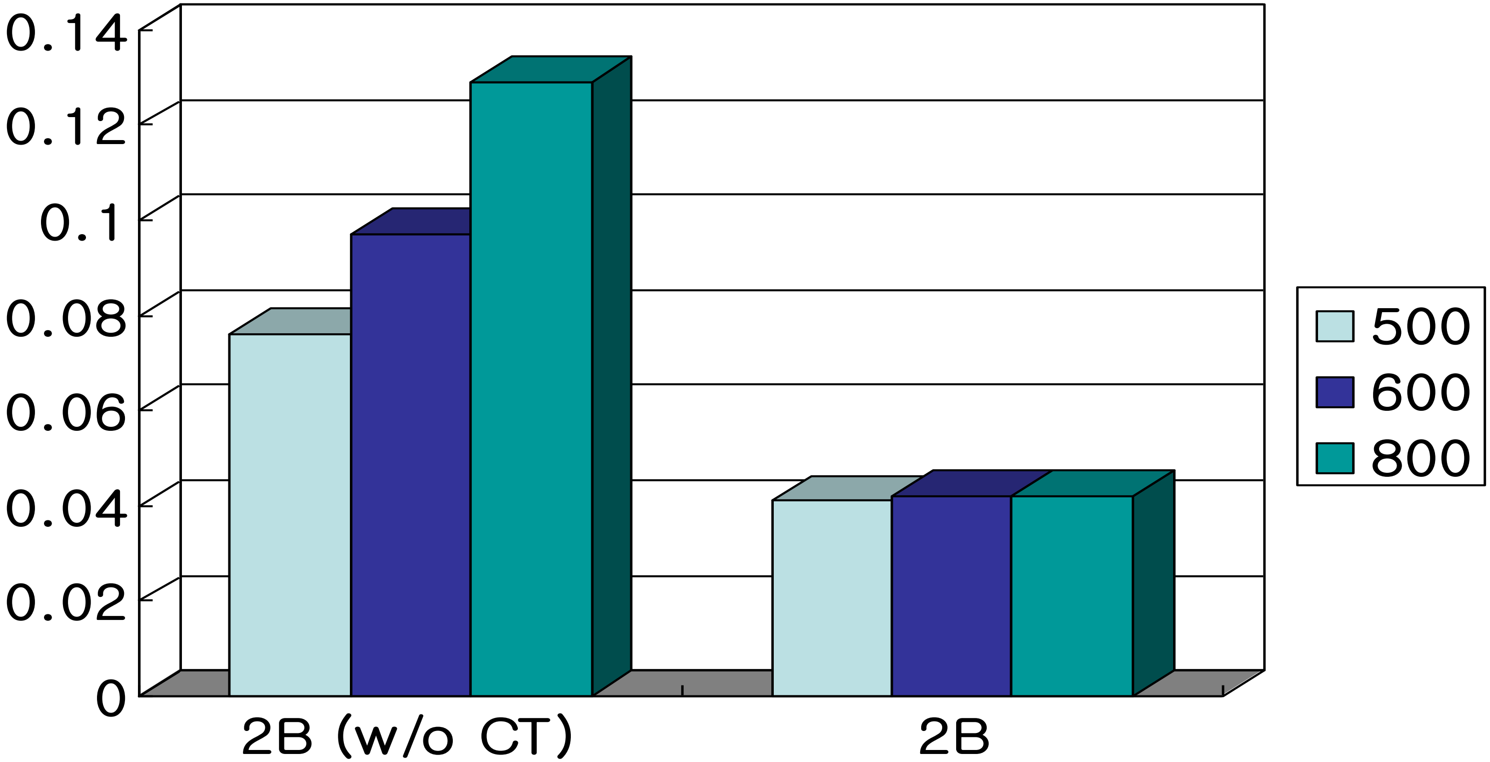
Λ (MeV)	$\hat{d}^R$	$\langle 1B \rangle$	$\langle 2B \rangle$
500	1.00	4.85	$0.076 - 0.035\,\hat{d}^R = 0.041$
600	1.78	4.85	$0.097 - 0.031\,\hat{d}^R = 0.042$
800	3.90	4.85	$0.129 - 0.022\,\hat{d}^R = 0.042$

with $\hat{d}^R$-term, Λ-dependence has gone !!!

the astro S-factor (at threshold)

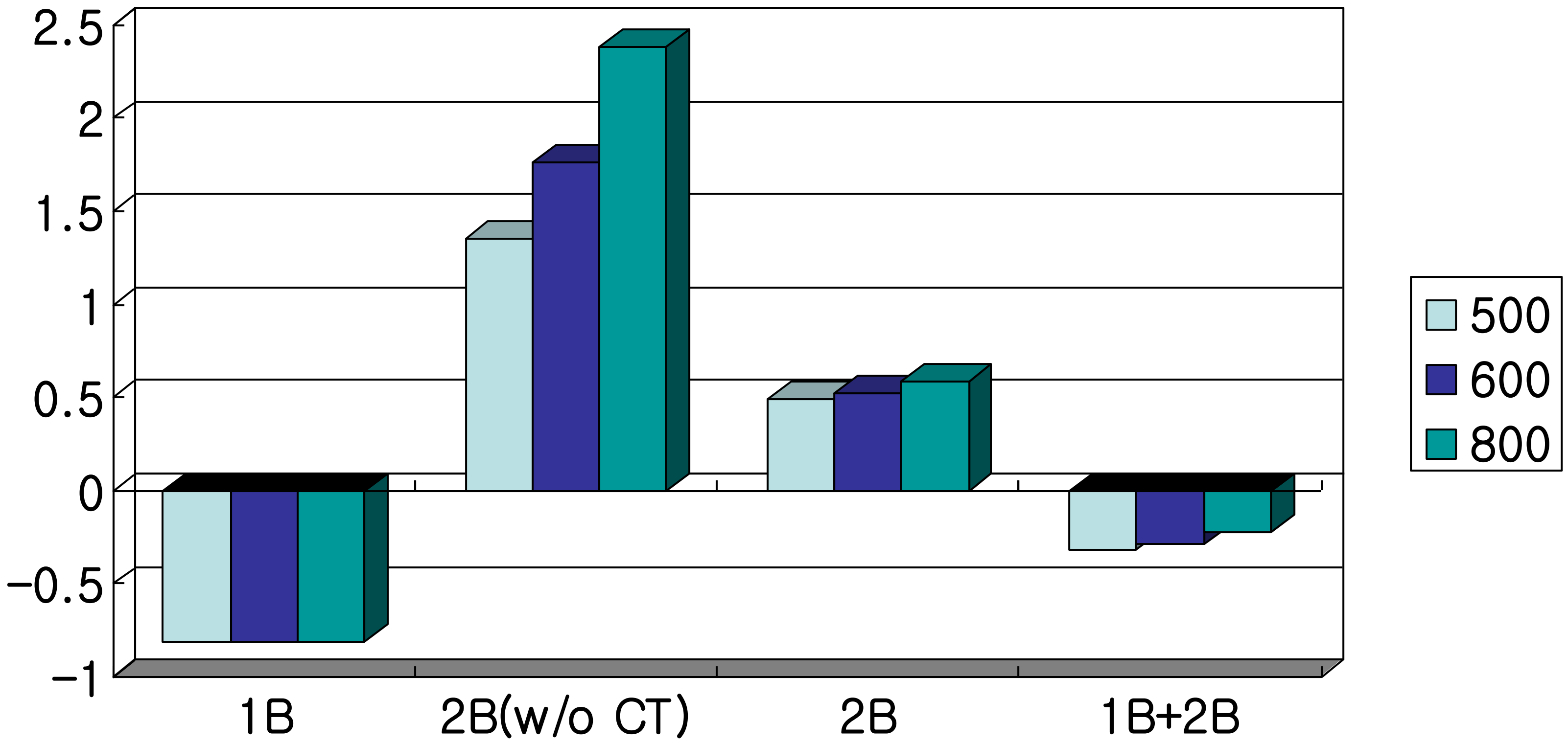
S_{pp}= 3.94 (1 ± 0.15 % ± 0.10 %) 10^{-25} MeV-barn

Results(*pp*)



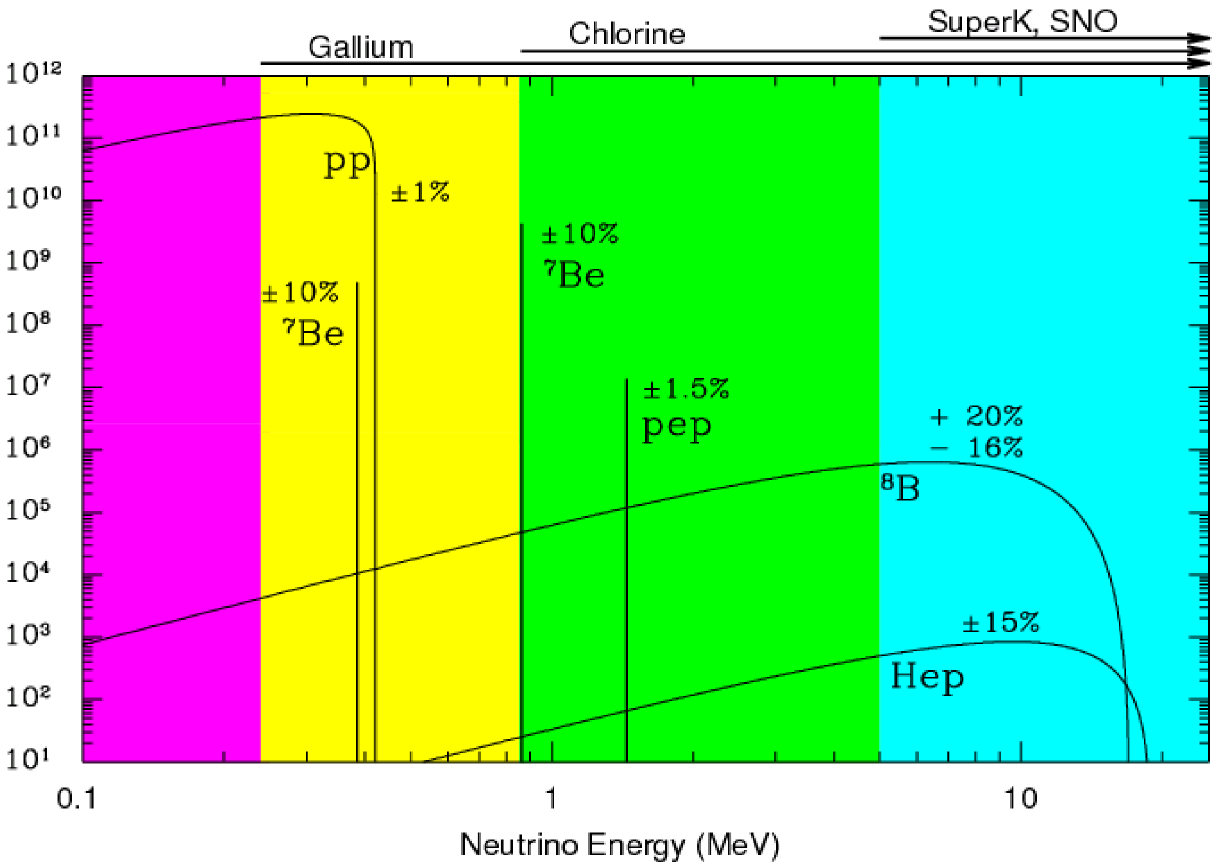
Results(hep: ${}^3He + p \rightarrow {}^4He + e^+ + \nu$)

Reduced matrix element with respect to Λ (MeV)



J. Bahcall said (hep-ex/0002018)

"... do not see any way at present to determine from experiment or first principle theoretical calculations a relevant, robust upper limit to the hep production cross section."



Discussions

- A few examples of EFT-description for EW probes have been reviewed, trying to explain
  - EFT can resolve very challenging questions in a light nuclei systems
  - Renormalization cures the mismatches in the short-range
  - Mismatches in long-ranged region is tough
- More efficient degrees of freedom ?
  - Pionless EFTs have scored huge successes
  - How about including heavy mesons ?

Thank you for your attention !